Communications

Equivalent Circuit of a Dipole Antenna Using Frequency-Independent Lumped Elements

Tee G. Tang, Quang M. Tieng, and Morris W. Gunn

***Abstract*—A four-element lumped-parameter equivalent circuit, consisting of a resistance, an inductance, and two capacitances, has been found to represent the feed-point impedance of a dipole antenna. The values of these elements are related only to the physical dimensions of the antenna, not the frequency of operation. Empirical formulas are given for all the elements. The equivalent circuit gives negligible errors in radiation resistance and reactance for dipole half-lengths less than 0.1λ, rising to 1% for resistance and 6% for reactance at 0.25λ. It can be readily used in standard computer software packages such as SPICE, PSPICE, and MICROCAP.**

I. Introduction

It is customary to represent the feed-point impedance of an electrically small dipole antenna by a lumped-element equivalent circuit consisting of a capacitance in series with a small frequency-dependent resistance (radiation resistance). This simple model has two distinct disadvantages: 1) the equivalent circuit becomes grossly inaccurate at or near the resonant frequency of the antenna; and 2) the value of the resistance has to be varied for different frequencies. The resonant frequency can be accounted for by inserting an inductance into the circuit. If this inductance is placed in series [1], the radiation resistance is still dependent on the frequency of operation.

Chu [2] has proposed an equivalent circuit for a dipole antenna based on a set of orthogonal spherical waves (or modes) of the antenna. This model makes use of L, C, and R elements and allows an indefinite expansion of the equivalent circuit to account for higher order modes.

A broad-band lumped-element equivalent circuit proposed by Streable and Pearson [3] is based on realizable "terminal eigenadmittances." It was shown in that paper, however, that the realizations for all but the lowest order terminal eigenadmittances become exceedingly complex.

When higher order modes are ignored, Chu's equivalent circuit is reduced to three elements while Streable and Pearson's equivalent circuit is reduced to four elements. However, it is found that both of these simplified circuits produce errors greater than 30% when compared against analytical solutions such as those given by Schelkunoff [4] or King-Middleton's approximation [5].

It is the purpose of this paper to report a more accurate four-element lumped-element equivalent circuit to represent the impedance of a dipole antenna of electrical half-length up to approximately 0.3 wavelength. This equivalent circuit will account for the first resonance of the dipole and can also be used at all frequencies below resonance. The equivalent circuit contains elements related to the physical dimensions of the antenna and independent of the frequency of operation.

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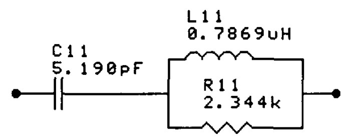


Fig. 1. Three-element equivalent circuit of a dipole [2] (h = 0.9 m, $a = 0.00264$ m; $h/a = 341$).

II. Equivalent Circuits Derived from Chu [2] and Streable and Pearson [3]

In practice, a dipole antenna is usually operated at or near its first resonant frequency. However, electrically small antennas, such as active receiving antennas, operate well below the resonant frequency. Thus, for these cases, higher order modes of resonance may be ignored. Two simpler equivalent circuits may then be deduced from [2] and [3]. These are, respectively

1. a three-element equivalent circuit from [2];
2. a four-element equivalent circuit from [3].

For a monopole antenna with an infinite groundplane, the values of L and R are halved while C is doubled.

A. The Three-Element Equivalent Circuit [2]

When higher order modes are ignored, Chu's equivalent circuit is reduced to that shown in Fig. 1. The values of L_{11}, C_{11}, and R_{11} will depend on the physical dimensions of the dipole antenna, independent of the frequency of operation. It is to be noted that R_{11} itself is not the radiation resistance. Their values are determined as follows.

The antenna feed-point impedance $Z_a = R_a + jX_a$ may be obtained from various sources or calculated using one of the classical antenna analysis methods. The value of C_{11} is determined from the reactance of the antenna at a frequency (f) much lower than the resonant frequency. In this process, it is assumed that L_{11} and R_{11} may be ignored, and C_{11} is given by

$$C_{11} = \frac{|X_a|}{2\pi f}. \tag{1a}$$

Alternatively, the value of C_{11} is related to the dipole half-length (h) and radius (a) by [4]

$$C_{11} = \frac{27.82 \times 10^{-12} h}{\ln(2h/a) - 1.693}. \tag{1b}$$

The inductance L_{11} and resistance R_{11} are estimated at the resonant frequency (ω_o) at which the reactance of the antenna vanishes and the radiation resistance is R_{ao}

$$L_{11} = \frac{1}{\omega_o^2 C_{11}} + C_{11} R_{ao}^2 \tag{1c}$$

$$R_{11} = \frac{L_{11}}{C_{11} R_{ao}}. \tag{1d}$$

It is found that in practice this equivalent circuit is reasonably accurate for the reactive component of the antenna impedance when the

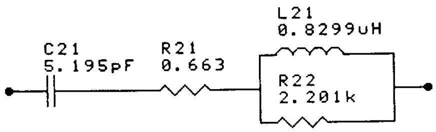


Fig. 2. Four-element equivalent circuit of a dipole [3] ($h = 0.9$ m; $a = 0.00264$ m; $h/a = 341$).

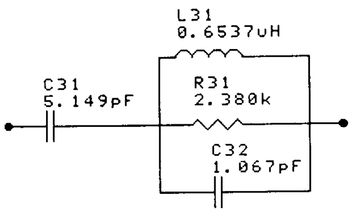


Fig. 3. New four-element equivalent circuit of a dipole ($h = 0.9$ m; $a = 0.00264$ m; $h/a = 341$).

antenna is electrically short, but grossly inaccurate for the resistive component. It tends to predict a higher radiation resistance below resonance.

B. The Four-Element Equivalent Circuit [3]

Similarly to the thee-element equivalent circuit, a four-element equivalent circuit results if higher order modes are ignored in Streable and Pearson's proposal. This is shown in Fig. 2. The values of R_{21}, R_{22}, C_{21}, and L_{21} are given by [3]

$$R_{21} = 0.663\Omega \tag{2a}$$

$$R_{22} = 2200.6\Omega \tag{2b}$$

$$C_{21} = \frac{0.00272l}{\pi c}F \tag{2c}$$

$$L_{21} = \frac{434.55l}{\pi c}H \tag{2d}$$

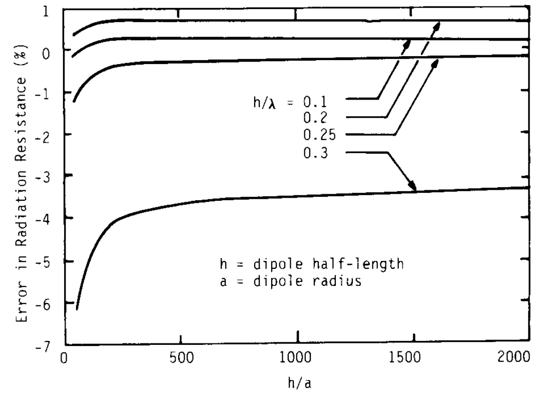
where $c = 3\times10^8$ (m/s); and $l = 2h$ =total length of dipole (m). It is noted that in the formulas above, the radius of the dipole antenna has not been included. By comparing the values of C_{11} [(1b)] and C_{21} [(2c)], it is found that (2c) is valid only for wires with $h/a = 341$.

The input impedance for a dipole with half-length $h = 0.9$ m calculated using (2a)–(2d) shows large errors when compared with the input impedance calculated using the King-Middleton quasi-zero approximation. The greatest error occurs when the frequency of operation is well below the resonant frequency and the radiation resistance is less than approximately 2 Ω. This is due to the presence of R_{21}, which sets the minimum value of the series radiation resistance.

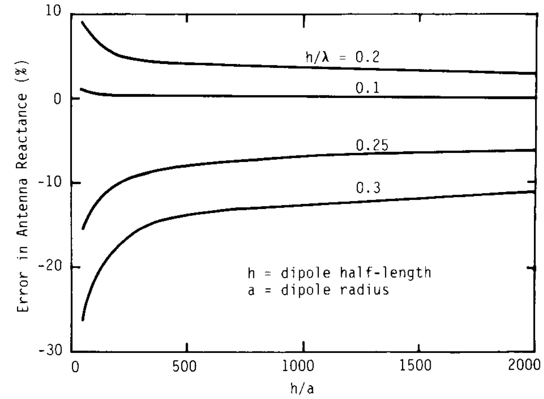
III. New Four-Element Equivalent Circuit

After an extensive empirical examination of many different L, C, and R combinations, the equivalent circuit of Fig. 3 was found to track the dipole impedance very closely. It consists of four elements and is of similar configuration to the three-element circuit of Chu [2], except for the added capacitance C_{32}.

The values of the elements in Fig. 3 are obtained by "point matching" the resulting impedance against the theoretical values from



(a)



(b)

Fig. 4. Accuracy of proposed equivalent circuit of a dipole: (a) radiation resistance of antenna; and (b) reactance of antenna.

the induced emf method. The resulting impedance of the proposed equivalent circuit coincide with the theoretical values at four points, comprising two on the radiation resistance and two on the antenna reactance curves. As an example, when the theoretical values from Wolff's equations [6, equations (3.120) and (3.174)] are used, the empirical equations for the four elements are as follows:

$$C_{31} = \frac{12.0674h}{\log(2h/a) - 0.7245}\text{pF} \tag{3a}$$

$$C_{32} = 2h\left\{\frac{0.89075}{[\log(2h/a)]^{0.8006} - 0.861} - 0.02541\right\}\text{pF} \tag{3b}$$

$$L_{31} = 0.2h\{[1.4813\log(2h/a)]^{1.012} - 0.6188\}\mu\text{H} \tag{3c}$$

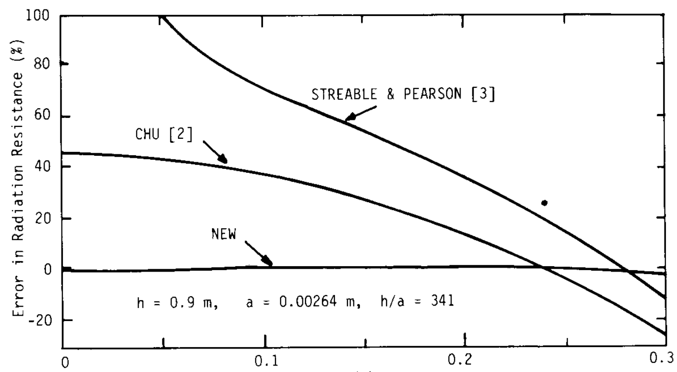
$$R_{31} = 0.41288[\log(2h/a)]^2 + 7.40754(2h/a)^{-0.02389} - 7.27408\text{k}\Omega. \tag{3d}$$

These empirical equations take into account the physical dimensions of the dipole, viz., half-length (h) and radius (a), which are expressed in meters.

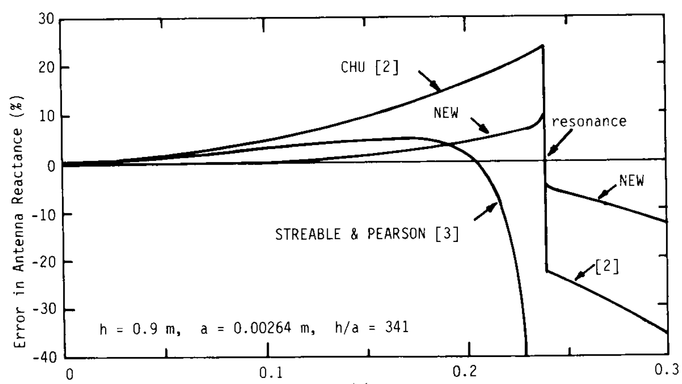
Fig. 4(a) and 4(b) shows the percentage errors of the antenna feed-point impedance as a function of h/a ratio for dipole half-lengths of 0.1, 0.2, 0.25, and 0.3 wavelength (λ). It can be seen that for

TABLE I
COMPARISON OF ANTENNA INPUT IMPEDANCES AGAINST WOLFF [6] (h =0.9 m; a =0.0026 4 m; h/a =341)

Normalized Length $2h/\lambda$	Frequency (MHz) f	Input Impedance (ohms) Fig. 1	Fig. 2	Fig. 3	Wolff [6]
0.125	20.83	4.52−j1369	6.02−j1361	3.15−j1397	3.15−j1394
0.250	41.67	17.96−j532	21.94−j519	13.50−j563	13.44−j568
0.375	62.50	40.04−j187	47.96−j171	34.27−j211	34.02−j218
0.500	83.33	70.25+j31.69	83.36+j51.23	72.94+j39.28	73.13+j42.35



(a)



(b)

Fig. 5. Comparison of three equivalent circuits: (a) radiation resistance of antenna; and (b) reactance of antenna.

TABLE II
COMPARISON OF ANTENNA INPUT IMPEDANCES AGAINST ELLIOTT [7]

Normalized Length 2h/λ	Normalized Radius a/λ	Input Impedance in ohms: Storer's two-term approximation	King-Middleton second-order approximation [7]	New equivalent circuit	Wolff [6]
0.25	0.01	11.63−j185	13.98−j166	13.49−j332	13.44−j337
0.25	0.0001	12.93−j723	12.90−j811	13.48−j886	13.44−j890
0.5	0.01	101.0+j32.82	92.51+j38.30	72.86+j38.23	73.13+j42.35
0.5	0.0001	80.15+j42.61	79.89+j43.47	72.99+j40.37	73.13+j42.35

$h/a > 100$ and $h < 0.25\lambda$, the errors in the radiation resistance are less than 1%, and the errors in the antenna reactance are less than 6%. For cases in which $h < 0.1\lambda$, the proposed equivalent circuit exhibits errors of less than 1% for $h/a > 50$.

Table I shows the typical antenna input impedances for the equivalent circuits of Figs. 1–3 for comparison purposes. It is obvious from the table that the values from Fig. 3 and Wolff [6] agree very well.

Fig. 5 compares the three types of equivalent circuit graphically and clearly demonstrates the superiority of the proposed new four-element equivalent circuit.

Table II compares the antenna input impedances of the new equivalent circuit against values given by Elliott [7, Table 7.5]. The general qualitative agreement is good.

IV. Conclusion

The feed-point impedance of a dipole or monopole antenna can be represented by a lumped-element equivalent circuit that has L, C, and R elements whose values are only related to the physical dimensions of the antenna, independent of the frequency of operation. A proposed four-element equivalent circuit is shown to have an error of less than 1% for the radiation resistance and 6% for the reactance, for operation up to the dipole's natural resonant frequency. Also, it is adequate to represent the impedance for dipole half-lengths up to approximately 0.3 wavelength.

The equivalent circuit can be readily used in standard computer software packages such as SPICE, PSPICE, and MICROCAP to analyze circuits containing dipole or monopole antennas with passive and active devices. It is particularly useful for analysis and design of electrically small nonresonant antenna elements such as used in active antennas and untuned frequency hopping antennas.

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Use of Equivalent Boundary Conditions for the Solution of a Class of Strip Grating Structures

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***Abstract*—Equivalent boundary conditions for an electromagnetic strip grating are used to derive reflection and transmission coefficients for a boundary value problem consisting of three nonconducting media where the strip grating exists at one of the planar boundaries. The results are used to generate data for a number of cases where data and/or theories exist. The theory presented here compares favorably with the literature cited for the limiting condition of the period of the grating small when compared with the free-space wavelength of the radiation.**

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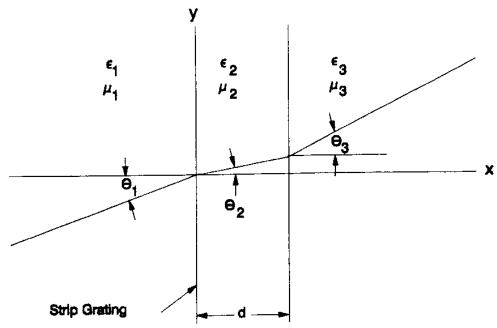

Fig. 1. A periodic strip grating mounted on a dielectric slab in free space.

I. Introduction

The development of equivalent boundary conditions for periodic surfaces is not new to the literature. Since this communication deals with the applications of these boundary conditions, reference is made to some of that literature [1]-[4] and to a short history of their development given in [5].

The use of equivalent boundary conditions for a periodic grating of ribbon conductors for solving boundary value problems is, to the author's knowledge, limited to cases where transmission and reflection coefficients [1] are derived for normal incidence on a grid in free space; where a computational study of strip grating antennas [6] was reported and closed-form analytical expressions were derived from which to calculate the resonance frequencies for certain cases of those antennas; and where equivalent boundary conditions [7] were used to predict surface waves along a metal grating on a grounded dielectric slab. It is the purpose of this paper to show that their use for a class of problems where the period of the grating is small compared with the free-space wavelength of the incident radiation is valid and fairly simple to implement.

II. Solution Procedure

Fig. 1 shows the boundary value problem of three dielectric regions of which regions 1 and 2 are separated by a strip grating of zero thickness, period p, gap width a, and strip width b. The incident and reflected waves in each region are at the angles shown relative to the normals of the interface planes. The boundary conditions at the two boundaries are that the tangential electric field intensity and the normal magnetic flux density are continuous. The tangential magnetic field intensity is continuous at the right boundary. At the left boundary the equivalent boundary conditions for the strips, which are oriented along the z axis, are

$$E_z = \frac{l_2}{2}\left[j\omega\mu_a(H_{y2} - H_{y1}) + \frac{1}{\epsilon_a}\frac{\partial}{\partial z}(\epsilon_2 E_{x2} - \epsilon_1 E_{x1})\right] \quad (1)$$

$$H_{z2} - H_{z1} = 2l_1\left(-j\omega\epsilon_a E_y + \frac{1}{\mu_a}\frac{\partial B_x}{\partial z}\right), \quad (2)$$

with

$$l_1 = \frac{p}{\pi}\ln\sec\frac{\pi b}{2p} = \frac{p}{\pi}\ln\csc\frac{\pi a}{2p} \quad (3)$$

$$l_2 = \frac{p}{\pi}\ln\csc\frac{\pi b}{2p} = \frac{p}{\pi}\ln\sec\frac{\pi a}{2p} \quad (4)$$