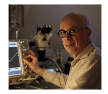
# Design of negative feedback amplifiers



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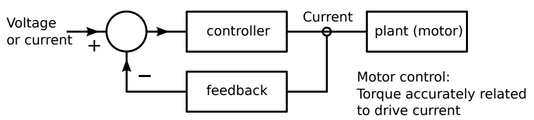
anton@montagne.nl
https://www.analog-electronics.eu

## Step by step orthogonal design of negative feedback amplifiers

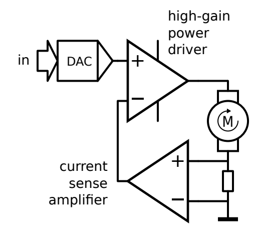
| Design step | Result | Design tool |
|---|---|---|
| *Setting up and understanding the specifications* | Function, performance, costs and environment | SLiCAP |
| Design considerations w.r.t. the amplifier type: A, B, C, D | Feedback configuration | SLiCAP |
| Design considerations w.r.t. the V/I drive capability and biasing stability | Biasing concept, supply voltages | SLiCAP |
| Design considerations w.r.t. the noise performance | Controller input noise sources | SLiCAP |
| Design considerations w.r.t. the mid-band accuracy and bandwidth | Loop gain and LP product | SLiCAP |
| Component selection (e.g. operational amplifier) | Putting it all together | Database |
| Frequency compensation | Frequency response, PZ pattern | SLiCAP |
| Verification of the weak nonlinearity | Distortion | LTspice |
| *(Specification of bias sources)* | *Design of bias sources* | SLiCAP / LTspice |

## Negative feedback current drivers

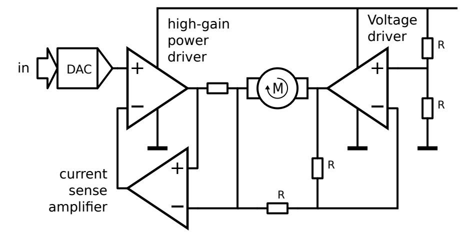



Fix parameter B or D:
- B and D: load current sensing: output series feedback; Zout = infinite.
- B: source voltage comparison: input series feedback; Zin = infinite.
- D: source current comparison: input parallel feedback; Zin = 0

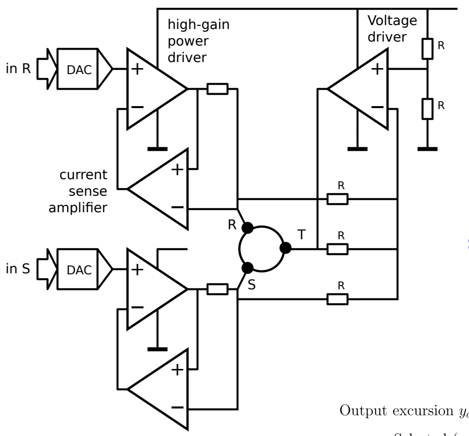
### Half-bridge single phase driver




### Full-bridge single phase driver




### Three-phase motor driver




## Performance aspects

### Static and dynamic drive capability

The static and dynamic drive capability of a feedback amplifier at best equal those of its controller.
Voltage drop across and current flow through feedback elements cause a reduction of the static and dynamic drive capability.

### Power efficiency

The power efficiency of a feedback amplifier at best equals that of its controller. Power losses and energy storage in feedback elements generally cause a reduction of the power efficiency.

### Noise

The equivalent-input noise sources of a feedback amplifier at best equal those of its controller.
Passive feedback elements generally increase the contribution of the controller input noise sources to the total source-referred noise.
Noise sources associated with passive and active feedback elements also cause deterioration of the signal-to-noise ratio.

### Zero error (offset) and drift

The (temperature-dependent) equivalent-input offset voltage and offset current of DC feedback amplifiers at best equal those of its controller.
Passive feedback elements generally increase the contribution of these sources to the total source-referred (zero error) offset.
Offset sources associated with active feedback elements also contribute to the total source-referred (zero error) offset.

### PSRR

The (voltage) PSRR of an amplifier can be modeled with an equivalent input voltage (noise) source, whose value equals the quotient of the power supply ripple and the PSRR. Coclusions for f eedback amplifiers correspond those given for noise.

### Static inaccuracy

The static gain inaccuracy of a feedback amplifier at best equals the reciprocal value of the static (DC) **loop gain**.
Generally the feedback network itself also contributes to the static gain inaccuracy.

### Static weak nonlinearity

The static differentiial gain error of a feedback amplifier at best equals the quotient of the static differentail gain error of the loop gain and the loop gain in the quiescent operating point.

### Bandwidth

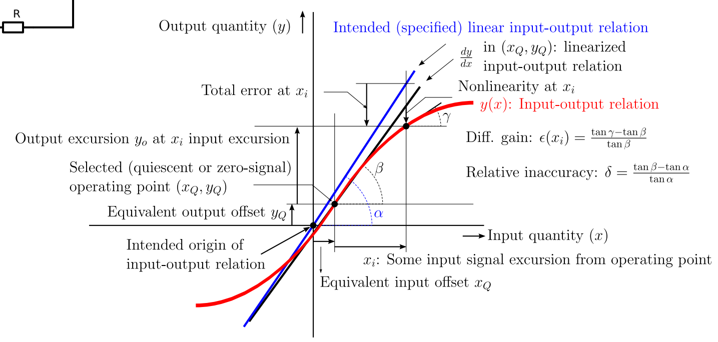
The bandwidth of a negative feedback amplifier equals that of its **servo function** (also: discrepancy factor).
For an all-pole feedback amplifier with **n dominant poles** it equals the n-th root of its n-th order l**oop gain-poles product**.

### Dynamic weak nonlinearity

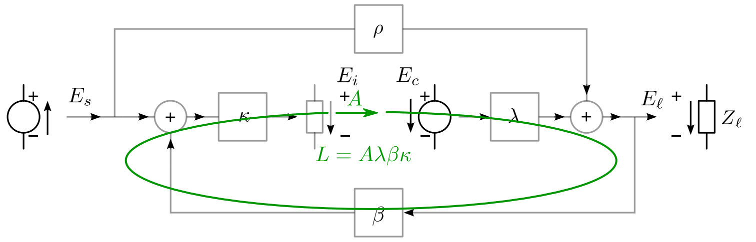
The frequency dependent differentiial gain error of a feedback amplifier at best equals the quotient of the frequency-dependent differentail gain error of the loop gain and the magnitude of the frequency dependent loop gain in the quiescent operating point.

## Input-output relation (definitions)

### Static if independent of rate of change




## Asymptotic-gain feedback model




$A$ : Loop gain reference variable: gain of the controlled source selected as loop gain reference

$\beta$ : Feedback factor (reciprocal value of ideal gain)

$\rho$ : Direct transfer

Asymptotic-gain: $A_{f\infty} = \lim_{A\to\infty} \frac{E_\ell}{E_s}$

Loop gain: $L = A\left.\frac{E_i}{E_c}\right|_{E_s=0}$

Gain: $A_f = \frac{E_\ell}{E_s} = A_{f\infty}\frac{-L}{1-L} + \frac{\rho}{1-L}$

Servo Function: $S = \frac{-L}{1-L}$ Measure for deviation of the gain from the asymptotic gain

Proper selection of the loop gain reference:
- Asymptotic-gain = ideal gain
- Direct transfer can be ignored
- Servo function is a measure for the difference between the actual gain and the ideal gain.

## Servo bandwidth with all-pole loop gain function

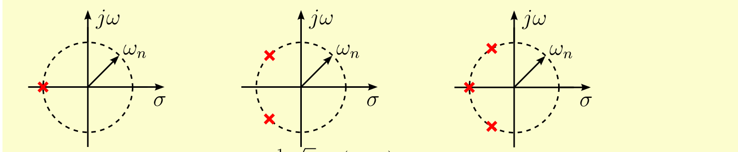
**All-pole loop gain function**
- Finite DC loop gain
- n poles

**All-pole servo function**
- Coefficient of the highest order of 's' in the denominator is the product of all the poles and the DC loop gain; (assume |L|>>1)

$$L(s) = \frac{L_{DC}}{\prod_{i=1}^{n}\left(1-\frac{s}{p_i}\right)}$$

$$S(s) = \frac{-L_{DC}}{1-L_{DC}}\frac{1}{1+\ldots+(-1)^n\frac{s^n}{(1-L_{DC})\prod_{i=1}^{n}p_i}}$$

**MFM or Butterworth filter response**




$p_1 = -\omega_n$ | $p_{1,2} = -\frac{1}{2}\sqrt{2}\omega_n(1\pm j)$ | $p_1 = -\omega_n$, $p_{2,3} = -\frac{1}{2}\omega_n \pm j\frac{1}{2}\sqrt{3}\omega_n$

$H_1(s) = \frac{1}{1+s/\omega_n}$ $H_2(s) = \frac{1}{1+s\sqrt{2}/\omega_n+s^2/\omega_n^2}$ $H_3(s) = \frac{1}{1+2s/\omega_n+2s^2/\omega_n^2+s^3/\omega_n^3}$

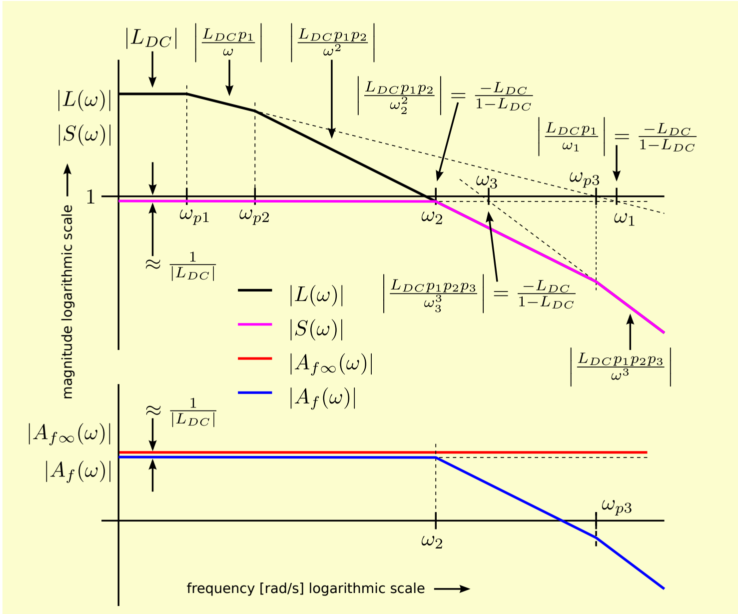
**-3dB bandwidth of n-th order MFM filter**

$$\omega_n = \sqrt[n]{\prod_{i=1}^{n}|p_i|}$$

**-3dB bandwidth of servo function with n-th order MFM filter characteristic**

$$\omega_n = \sqrt[n]{\left|(1-L_{DC})\prod_{i=1}^{n}p_i\right|} \approx \sqrt[n]{\left|L_{DC}\prod_{i=1}^{n}p_i\right|}$$

**Loop gain-poles product (LP product)**
- The bandwidth requirement for the amplifier sets a requirement for the product of the DC loop gain and the dominant poles of the loop gain.
- The controller should be selected or designed such that its contribution to this product is sufficiently large, while the number of poles should be kept as small as possible.




1. Dominant poles are those that contribute to the bandwidth of the servo function
2. The bandwidth of the servo function can be approximated as the frequency of intersection of the asymptotic approximation of the magnitude characteristic of the loop gain and unity (assuming dominant poles only)

**Procedure for finding dominant poles** (all-pole loop gain only)
1. Rank poles of the loop gain (ascending order of absolute frequency)
2. For increasing order i, calculate the achievable MFM bandwidth from the i-th order LP product
   - If the bandwidth decreases with the order i, or if the frequency of the i-th pole exceeds the (i-1) order MFM bandwidth the i-th pole is not dominant and the (i-1) order MFM bandwidth is the maximum achievable MFM bandwidth.

**Non dominant poles**
The influence of the non-dominant poles on the achievable MFM bandwidth cannot always be ignored.

## Frequency stability

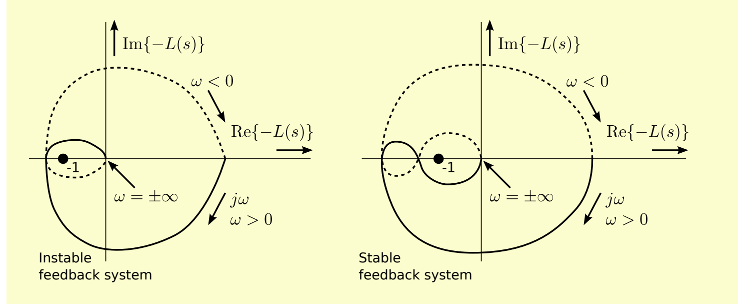
After the bandwidth has been designed, the amplifier may be instable, and in general will not have the desired frequency response.
A system is stable if all the roots of the characteristic equation (poles) are located in the left half of the complex plane. Note: a delay function has an infinite number of poles and zeros.

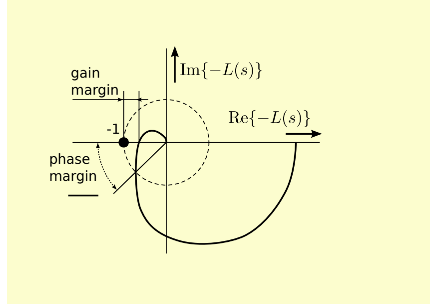
**Routh-Hurwitz stability criterion**
- Mathemathical test using the Routh Array

**Nyquist stability criterion**
- The number of clockwise encirclements of the point (-1,0) in the complex plane by the polar (contour) plot of -L equals the number of right half plane poles of the gain minus the number of right half plane poles of the loop gain

**Gain margin and phase margin**
Simple stability criterion for all-pole second-order feedback systems

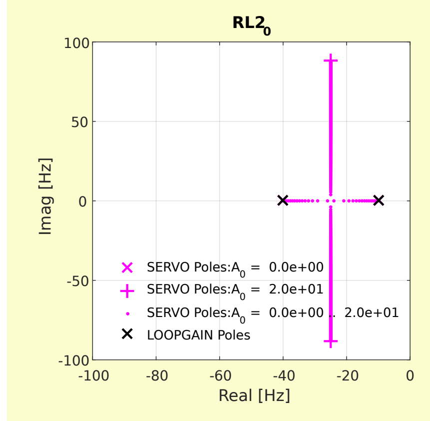
- The distance of the contour to -1 should be kept sufficiently large.
- 2-nd order MFM: PM = 60 degrees
- No unique correspondence between frequency response and gain and phase margin in higher order systems or in feedback systems that have zeros in the loop gain.



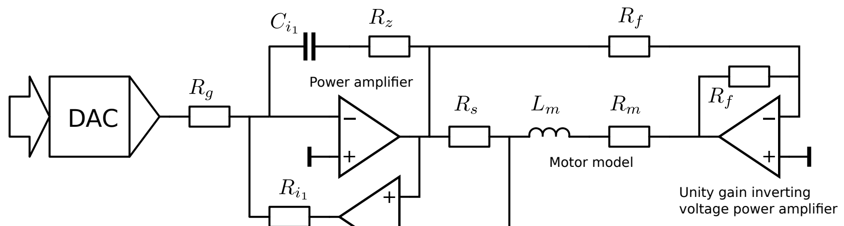

**Root-locus technique**
Graphical method, tracing out the paths (branches) of the poles of the servo function while increasing the DC or mid-band loop gain from zero to infinity.
1. The number of branches equals the number of poles
2. Poles are either real or complex conjugated
3. A branch starts at a pole of the loop gain
4. A branch ends at a zero of the loop gain. If there are n poles and m zeros, we assume m-n zeros at infinity
5. Parts of the real axis left from an odd number of poles plus zeros are part of a branch
6. If there are n poles and n zeros n-m branches go to infinity. The angle of their asymptotes is
   $$\theta_i = \frac{2i+1}{n-m}\pi,\ i = 0, 1, 2, \ldots$$
7. The asymptotes intersect the real axis at
   $$\sigma = \frac{\sum_{k=1}^{n}p_k - \sum_{i=1}^{m}z_i}{n-m}$$
8. Break away and arrival points on the real axis are found from solving $\frac{d}{ds}L(s) = 0$
9. The angles between the branches at the break-away or arrival points are equally spaced over 360 degrees
10. Each point on a branch satisfies |L(s)|=1 and arg(-L(s))=180 degrees
    Note: L=-Hk; difference between Black's model and the asymptotic gain model.

From the root locus plots we can learn that all-pole, higher order, negative feedback systems may have poles in the right half plan and be instable. Insertion of zeros in the left half plane may be benificial for the stability.




With SLiCAP we can draw root-locus plots with an arbitrarily selected root locus variable

## Example bandwidth design full-bridge current driver




Pole-zero compensation with PI controller:

$$\frac{L_m}{R_m+R_s} = C_{i_1}R_z$$

First-order system with PI controller:

$$B_1 = \frac{1}{2\pi}\frac{2R_sA_d}{R_{i_1}C_{i_1}(R_s+R_m)}$$

Second-order system with PII controller:

$$B_2 = \frac{1}{2\pi}\sqrt{\frac{2R_sA_d}{R_{i_1}R_{i_2}C_{i_1}C_{i_2}(R_s+R_m)}}$$